

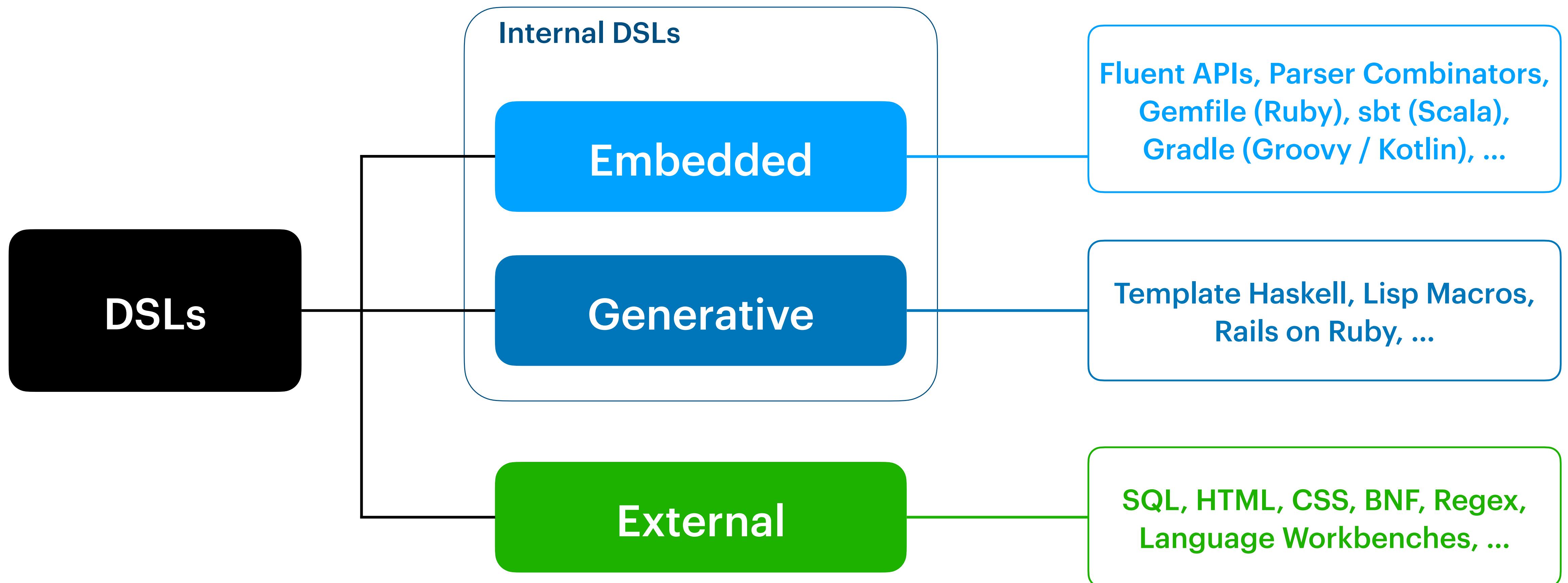


# Compositional Embeddings of Domain-Specific Languages

Yaozhu Sun, Utkarsh Dhandhania, Bruno C. d. S. Oliveira

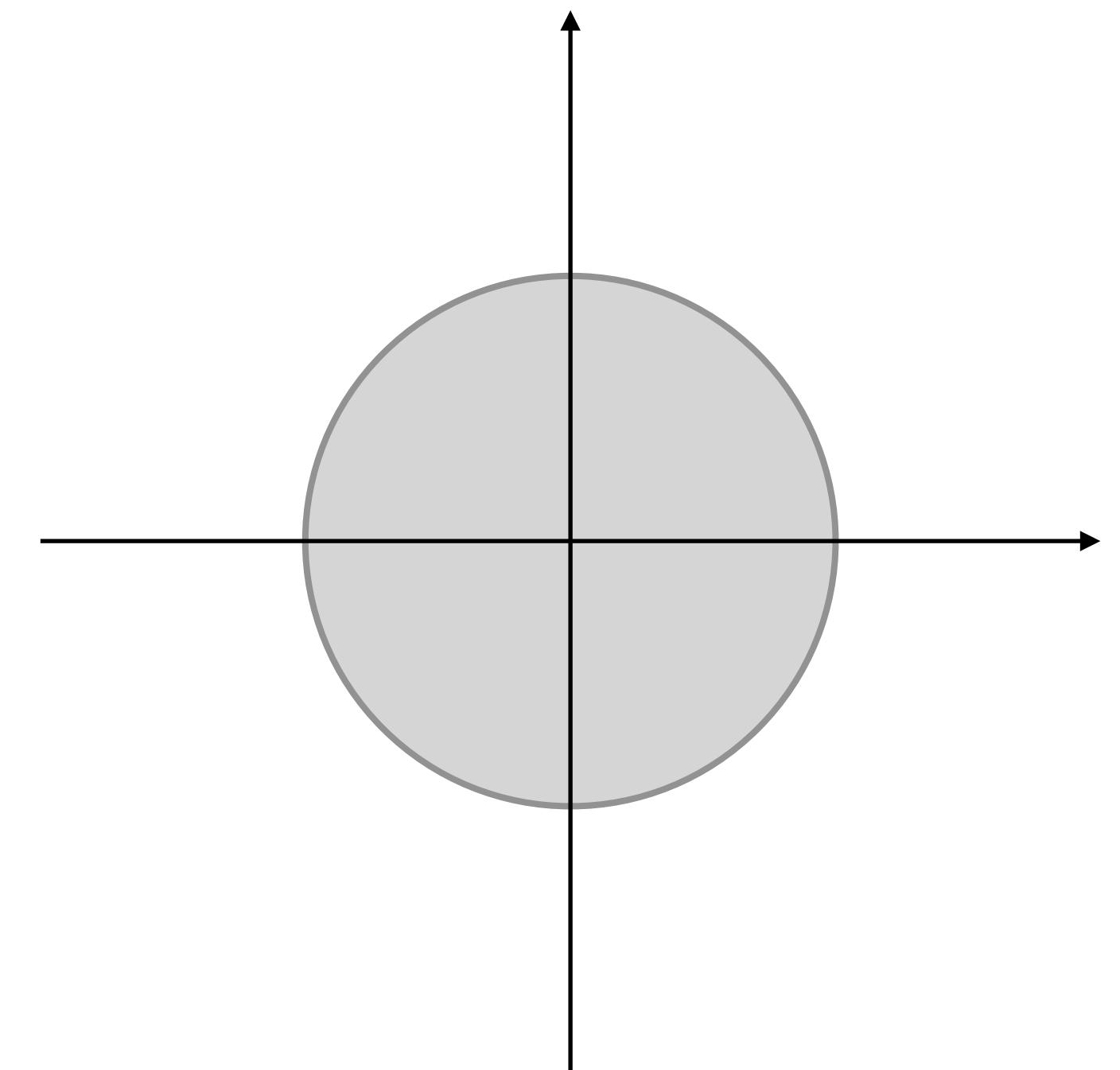
10 December 2022

# Taxonomy



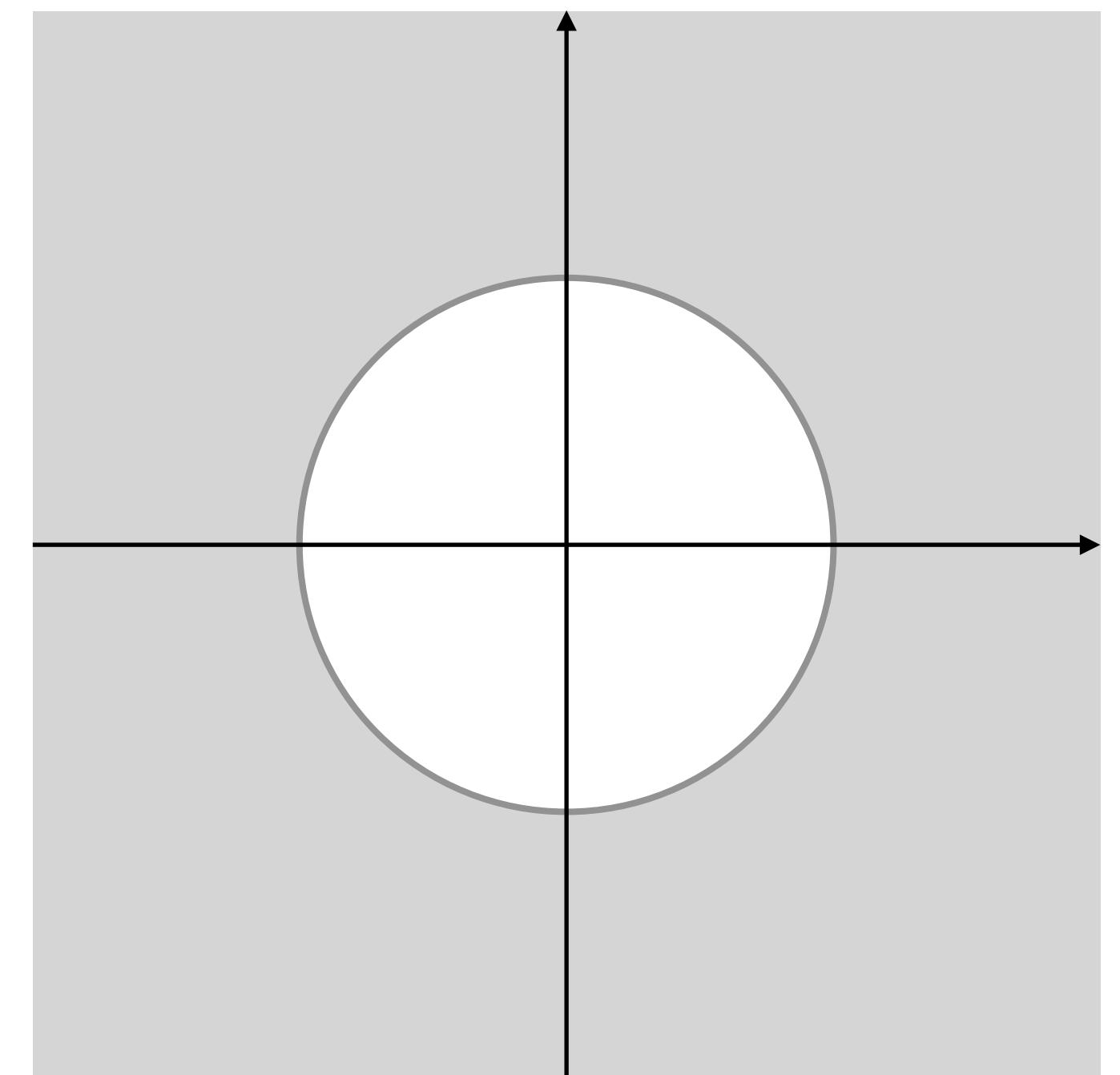
# An Embedded DSL for Regions

- (circle r) a circular region of radius r



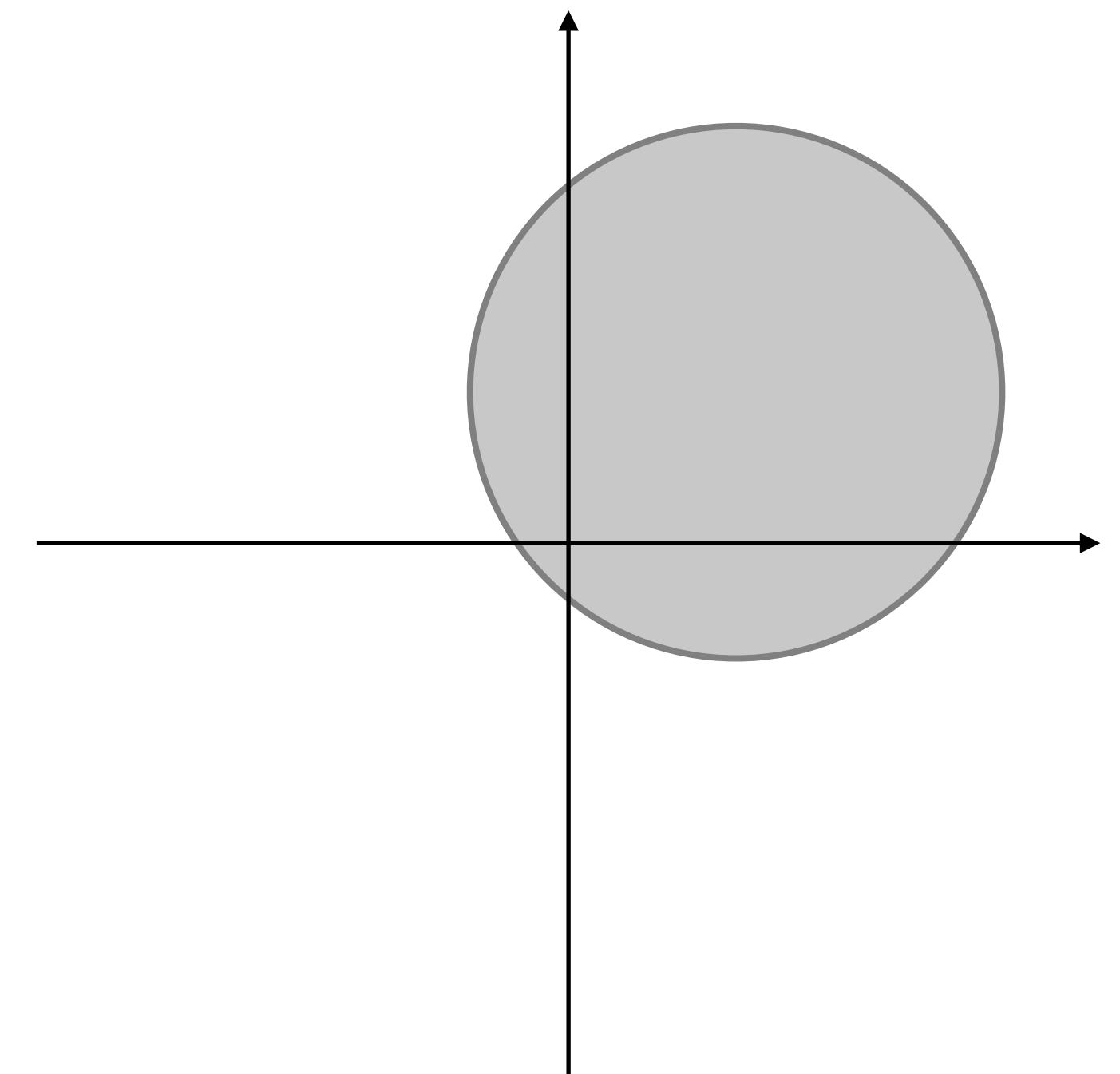
# An Embedded DSL for Regions

- (circle r) a circular region of radius r
- (outside a) the complement of region a



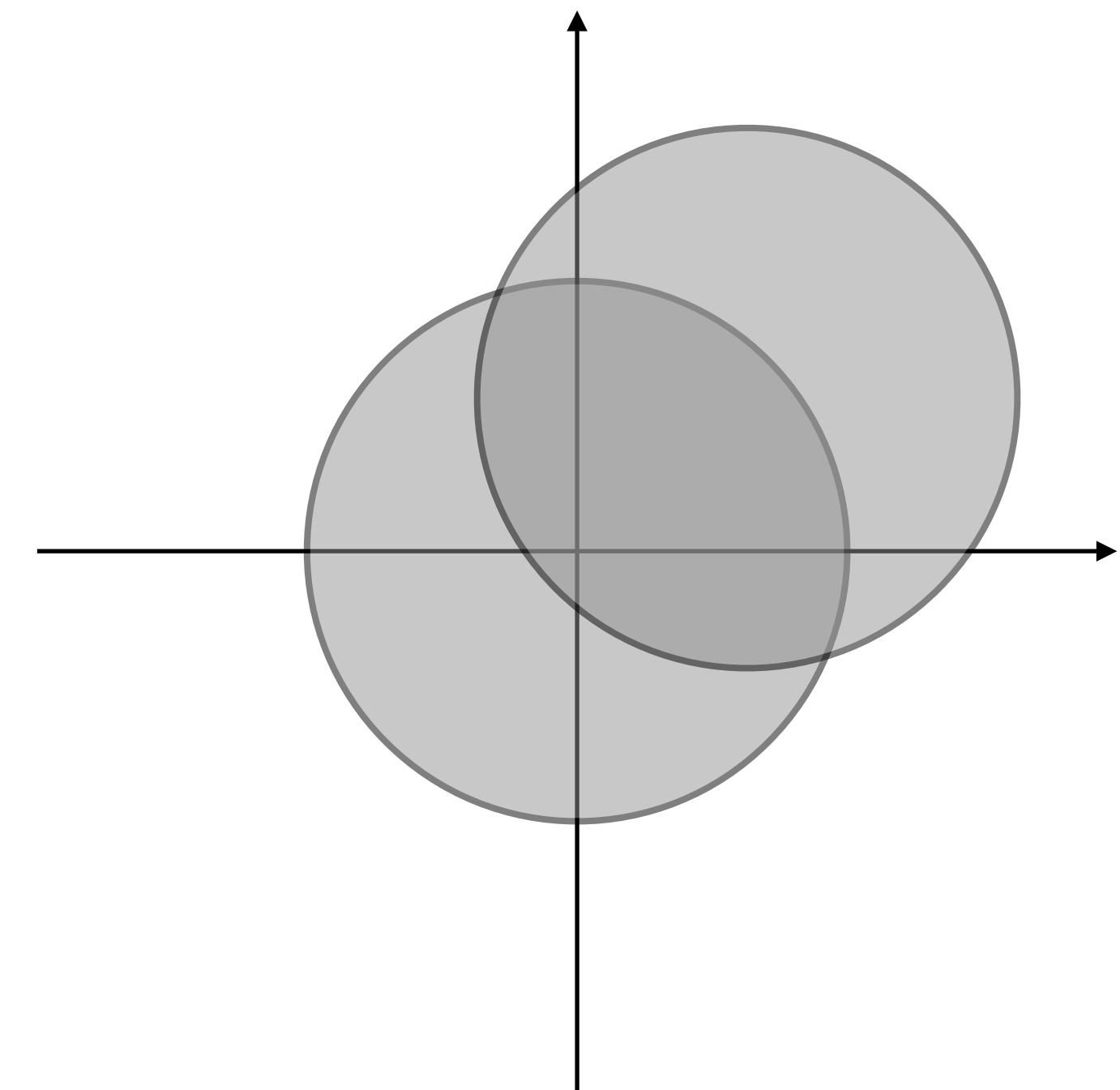
# An Embedded DSL for Regions

- (`circle r`) a circular region of radius  $r$
- (`outside a`) the complement of region  $a$
- (`translate v a`) translate  $a$  by  $v$



# An Embedded DSL for Regions

- (`circle r`) a circular region of radius  $r$
- (`outside a`) the complement of region  $a$
- (`translate v a`) translate  $a$  by  $v$
- (`union a b`) the union of  $a$  and  $b$
- (`intersection a b`) the intersection of  $a$  and  $b$



# Shallow

```
type Region = Int
```

syntactic size  
(number of AST nodes)

```
circle    :: Double → Region
circle      _ = 1
outside   :: Region → Region
outside     a = a + 1
union     :: Region → Region → Region
union      a b = a + b + 1
intersect :: Region → Region → Region
intersect a b = a + b + 1
translate  :: Vector → Region → Region
translate _ a = a + 1
```

# Shallow

vs.

# Deep

```
type Region = Int  
  
circle    :: Double → Region  
circle _ = 1  
outside   :: Region → Region  
outside a = a + 1  
union     :: Region → Region → Region  
union a b = a + b + 1  
intersect :: Region → Region → Region  
intersect a b = a + b + 1  
translate  :: Vector → Region → Region  
translate _ a = a + 1
```

syntactic size  
(number of AST nodes)

```
data Region where  
  Circle    :: Double → Region  
  Outside   :: Region → Region  
  Union     :: Region → Region → Region  
  Intersect :: Region → Region → Region  
  Translate :: Vector → Region → Region  
  
size :: Region → Int  
size (Circle _) = 1  
size (Outside a) = size a + 1  
size (Union a b) = size a + size b + 1  
size (Intersect a b) = size a + size b + 1  
size (Translate _ a) = size a + 1
```

# Linguistic Reuse

## (Shallow)

```
circles = go 20 (2 ** 18)
where go :: Int → Double → Region
      go 0 offset = circle 1
      go n offset = let shared = go (n - 1) (offset / 2)
                     in union (translate Vector { x = -offset, y = 0 } shared)
                           (translate Vector { x = offset, y = 0 } shared)
```

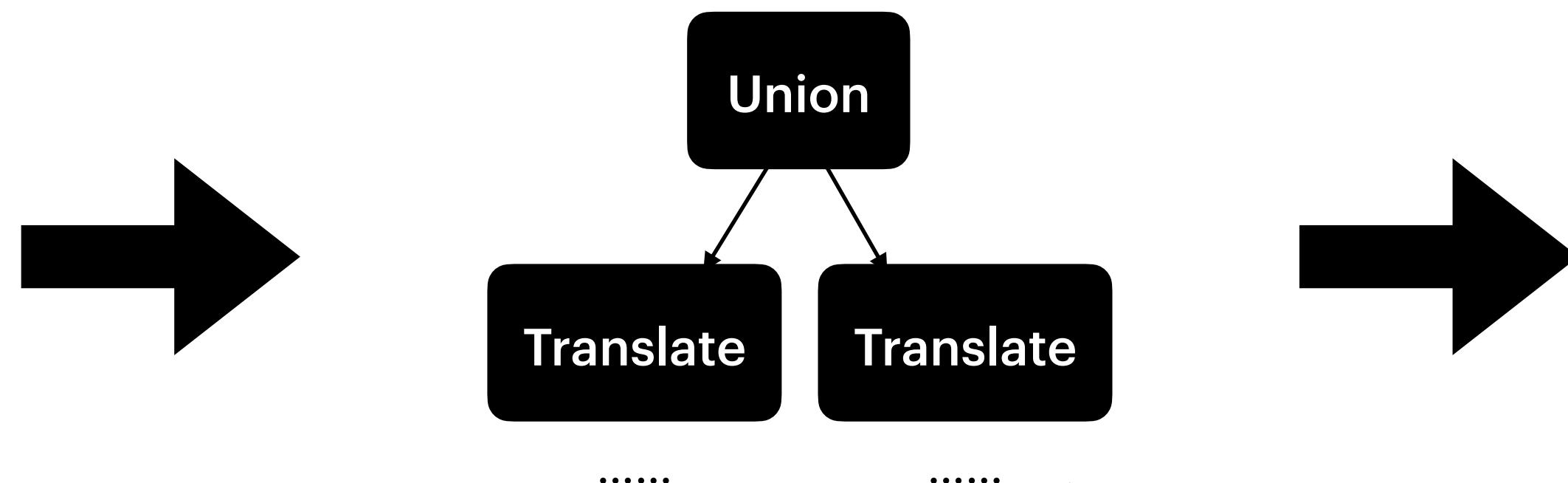
duplicate computation  
avoided (very fast!)

# Linguistic Reuse (Shallow)

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                           (translate Vector { x = offset, y = 0 } shared)
```

duplicate computation avoided (very fast!)

(Deep)  
AST



duplicate  
computation  
(very slow!)

# The Expression Problem

(Shallow) new language constructs

```
univ :: Region  
univ = 1
```

```
empty :: Region  
empty = 1
```

```
scale :: Vector → Region → Region  
scale _ a = a + 1
```

(Deep) ☹

# The Expression Problem

(Shallow) new language constructs

```
univ :: Region  
univ = 1
```

```
empty :: Region  
empty = 1
```

```
scale :: Vector → Region → Region  
scale _ a = a + 1
```

(Deep) ☹

(Deep) new semantic interpretations

```
contains :: Region → Vector → Bool  
Circle r `contains` p = p.x ** 2 + p.y ** 2 ≤ r ** 2  
Outside a `contains` p = not (a `contains` p)  
Union a b `contains` p = a `contains` p || b `contains` p  
Intersect a b `contains` p = a `contains` p && b `contains` p  
Translate Vector {..} a `contains` p = a `contains` Vector { x = p.x - x, y = p.y - y }
```

(Shallow) ☹

# Dependencies without Code Duplication

## (Deep)

<pre>isUniv :: Region → Bool isUniv Univ           = True isUniv (Outside a)   = isEmpty a isUniv (Union a b)   = isUniv a    isUniv b isUniv (Intersect a b) = isUniv a &amp;&amp; isUniv b isUniv (Translate _ a) = isUniv a isUniv (Scale _ a)   = isUniv a isUniv _              = False</pre>	<pre>isEmpty :: Region → Bool isEmpty Empty          = True isEmpty (Outside a)   = isUniv a isEmpty (Union a b)   = isEmpty a &amp;&amp; isEmpty b isEmpty (Intersect a b) = isEmpty a    isEmpty b isEmpty (Translate _ a) = isEmpty a isEmpty (Scale _ a)   = isEmpty a isEmpty _              = False</pre>
--	---

# Dependencies without Code Duplication

## (Deep)

<pre>isUniv :: Region → Bool isUniv Univ          = True isUniv (Outside a) = isEmpty a isUniv (Union a b) = isUniv a    isUniv b isUniv (Intersect a b) = isUniv a &amp;&amp; isUniv b isUniv (Translate _ a) = isUniv a isUniv (Scale _ a) = isUniv a isUniv _             = False</pre>	<pre>isEmpty :: Region → Bool isEmpty Empty        = True isEmpty (Outside a) = isUniv a isEmpty (Union a b) = isEmpty a &amp;&amp; isEmpty b isEmpty (Intersect a b) = isEmpty a    isEmpty b isEmpty (Translate _ a) = isEmpty a isEmpty (Scale _ a) = isEmpty a isEmpty _             = False</pre>
--	--

## (Shallow) non-modular workaround (code duplication needed)

(isUniv, isEmpty)

```
type Region = (Bool, Bool)  
  
univ      = (True,  False)  
empty     = (False, True)  
circle    = (False, False)
```

isEmpty a      isUniv a

```
outside a       = (snd a,           fst a)  
union a b       = (fst a || fst b, snd a && snd b)  
intersect a b   = (fst a && fst b, snd a || snd b)  
translate _ a   = (fst a,           snd a)  
scale _ a       = (fst a,           snd a)
```

# The 3rd Dimension of the EP

	FP	OOP	POLY.	COMP.
New constructs	○	●	●	●
New functions	●	○	●	●
<i>Dependencies</i>	●	●	○	●

● no code duplication      ○ code duplication needed

POLY. = Polymorphic embeddings, tagless-final embeddings, and other modern embeddings  
COMP. = Compositional embeddings

**“The holy grail is to combine the advantages of shallow and deep in a single implementation.”**

— Josef Svenningsson and Emil Axelsson

# Compositional Programming

# Compositional Embeddings

(written in the CP language)

```
type Vector = { x : Double; y : Double };
```

sort

```
type HudakSig<Region> = {
    Circle      : Double → Region;
    Outside     : Region → Region;
    Union       : Region → Region → Region;
    Intersect   : Region → Region → Region;
    Translate   : Vector → Region → Region;
}
```

l.

constructors

```
data Region where
    Circle      :: Double → Region
    Outside     :: Region → Region
    Union       :: Region → Region → Region
    Intersect   :: Region → Region → Region
    Translate   :: Vector → Region → Region
```

# Compositional Embeddings

(written in the CP language)

```
type Vector = { x : Double; y : Double };  
sort
```

```
type HudakSig<Region> = {  
    Circle : Double → Region;  
    Outside : Region → Region;  
    Union : Region → Region → Region;  
    Intersect : Region → Region → Region;  
    Translate : Vector → Region → Region;
```

l.

constructors

```
data Region where  
    Circle :: Double → Region  
    Outside :: Region → Region  
    Union :: Region → Region → Region  
    Intersect :: Region → Region → Region  
    Translate :: Vector → Region → Region
```

```
type Size = { size : Int };
```

```
sz = trait implements HudakSig<Size> ⇒ {  
    (Circle _).size = 1;  
    (Outside a).size = a.size + 1;  
    (Union a b).size = a.size + b.size + 1;  
    (Intersect a b).size = a.size + b.size + 1;  
    (Translate _ a).size = a.size + 1;  
};
```

method patterns

```
size :: Region → Int  
size (Circle _) = 1  
size (Outside a) = size a + 1  
size (Union a b) = size a + size b + 1  
size (Intersect a b) = size a + size b + 1  
size (Translate _ a) = size a + 1
```

instantiates  
sort Region

# Linguistic Reuse

```
sharing Region = trait [self : RegionSig<Region>] ⇒ {  
    circles = letrec go (n:Int) (offset:Double) : Region =  
        if n = 0 then Outside (Outside (Circle 1.0))  
        else let shared = go (n-1) (offset/2.0)  
            in Union (Translate { x = -offset; y = 0.0 } shared)  
                (Translate { x = offset; y = 0.0 } shared)  
    in let n = 20 in go n (pow 2.0 (n-2));  
};
```

duplicate computation  
avoided (very fast!)

# New Constructs & Interpretations

```
type HoferSig<Region> = {
    Univ : Region;
    Empty : Region;
    Scale : Vector → Region → Region;
};

sz' = trait implements HoferSig<Size> ⇒ {
    (Univ     ).size = 1;
    (Empty    ).size = 1;
    (Scale _ a).size = a.size + 1;
};

type RegionSig<Region> =
    HudakSig<Region> & HoferSig<Region>;
```

intersection type

```
type Eval = { contains : Vector → Bool };
eval = trait implements RegionSig<Eval> ⇒ {
    (Circle      r).contains p = pow p.x 2 + pow p.y 2 ≤ pow r 2;
    (Outside     a).contains p = not (a.contains p);
    (Union        a b).contains p = a.contains p || b.contains p;
    (Intersect    a b).contains p = a.contains p && b.contains p;
    (Translate {..} a).contains p = a.contains { x = p.x - x;
                                                y = p.y - y };

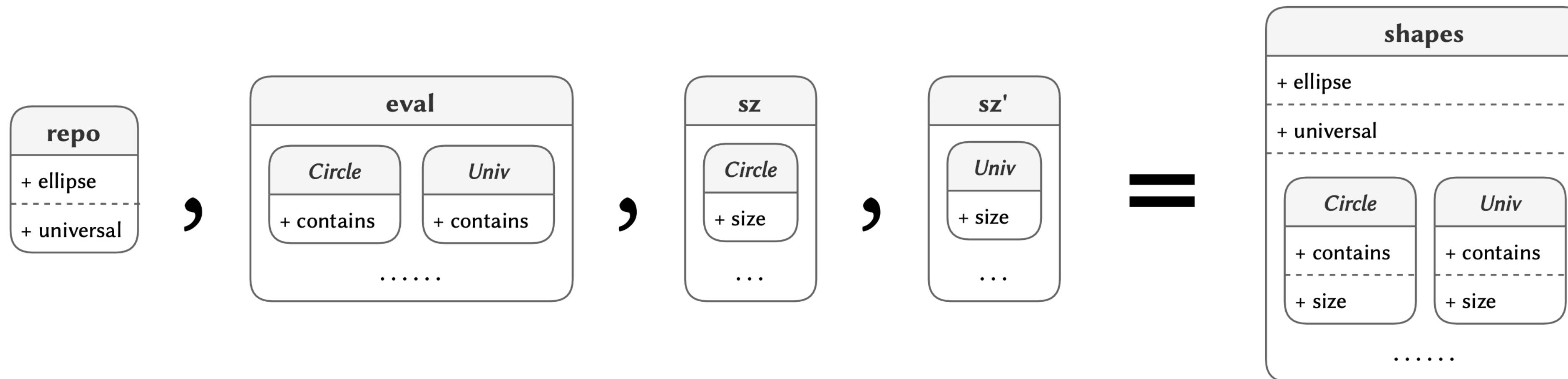
    (Univ         ).contains _ = true;
    (Empty        ).contains _ = false;
    (Scale {..} a).contains p = a.contains { x = p.x / x;
                                                y = p.y / y };
};

shapes = new repo @Eval&Size , eval , sz , sz' ;
```

nested trait composition

# Nested Trait Composition

```
shapes = new repo @Eval&Size , eval , sz , sz' ;  
shapes = new ((repo @Eval&Size)) , eval , sz , sz');
```



# Dependency Injection

## (Interface Type Refinement)

```
type IsUniv = { isUniv : Bool };  
  
chkUniv = trait  
  implements RegionSig<IsEmpty ⇒ IsUniv> ⇒ {  
    (Univ      ).isUniv = true;  
    (Outside    a).isUniv = a.isEmpty;  
    (Union     a b).isUniv = a.isUniv || b.isUniv;  
    (Intersect a b).isUniv = a.isUniv && b.isUniv;  
    (Translate _ a).isUniv = a.isUniv;  
    (Scale     _ a).isUniv = a.isUniv;  
    _           .isUniv = false;  
};
```

input      output

```
type IsEmpty = { isEmpty : Bool };  
  
chkEmpty = trait  
  implements RegionSig<IsUniv ⇒ IsEmpty> ⇒ {  
    (Empty      ).isEmpty = true;  
    (Outside    a).isEmpty = a.isUniv;  
    (Union     a b).isEmpty = a.isEmpty && b.isEmpty;  
    (Intersect a b).isEmpty = a.isEmpty || b.isEmpty;  
    (Translate _ a).isEmpty = a.isEmpty;  
    (Scale     _ a).isEmpty = a.isEmpty;  
    _           .isEmpty = false;  
};
```

# Comparison of Embeddings

	SHALLOW	DEEP	HYBRID	POLY.	COMP.
Transcoding free	●	●	○	●	●
Linguistic reuse	●	○	●	●	●
Language construct extensibility	●	○	○ <sup>1</sup>	●	●
Interpretation extensibility	○	●	●	●	●
Transformations and optimizations	○	●	●	○ <sup>2</sup>	○ <sup>2</sup>
Linguistic reuse after transformations	n/a	○	○	●	●
Modular dependencies	○	○ <sup>3</sup>	○ <sup>3</sup>	○	●
Nested pattern matching	○	●	●	○	○ <sup>4</sup>

<sup>1</sup> The extensibility of language constructs is limited or precludes exhaustive pattern matching.

<sup>2</sup> Transformations require some ingenuity and are sometimes awkward to write.

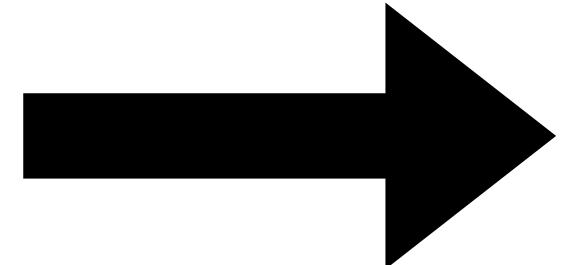
<sup>3</sup> Dependencies do not require code duplication but still refer to concrete implementations.

<sup>4</sup> Nested pattern matching is implemented as delegated method patterns.

# ExT

## (A Compositionally Embedded DSL for Document Authoring)

```
\Section[
    Welcome to \Href("https://plground.org")[PLGround]!
]
\Bold[CP] is a \Emph[compositional] programming
language. \\ There are \((1+1+1+1)\) key concepts in CP:
\Itemize[
    \Item[Compositional interfaces;]
    \Item[Compositional traits;]
    \Item[Method patterns;]
    \Item[Nested trait composition.]
]
```



### **WELCOME TO PLGROUND!**

**CP** is a *compositional* programming language.

There are 4 key concepts in CP:

- Compositional interfaces;
- Compositional traits;
- Method patterns;
- Nested trait composition.

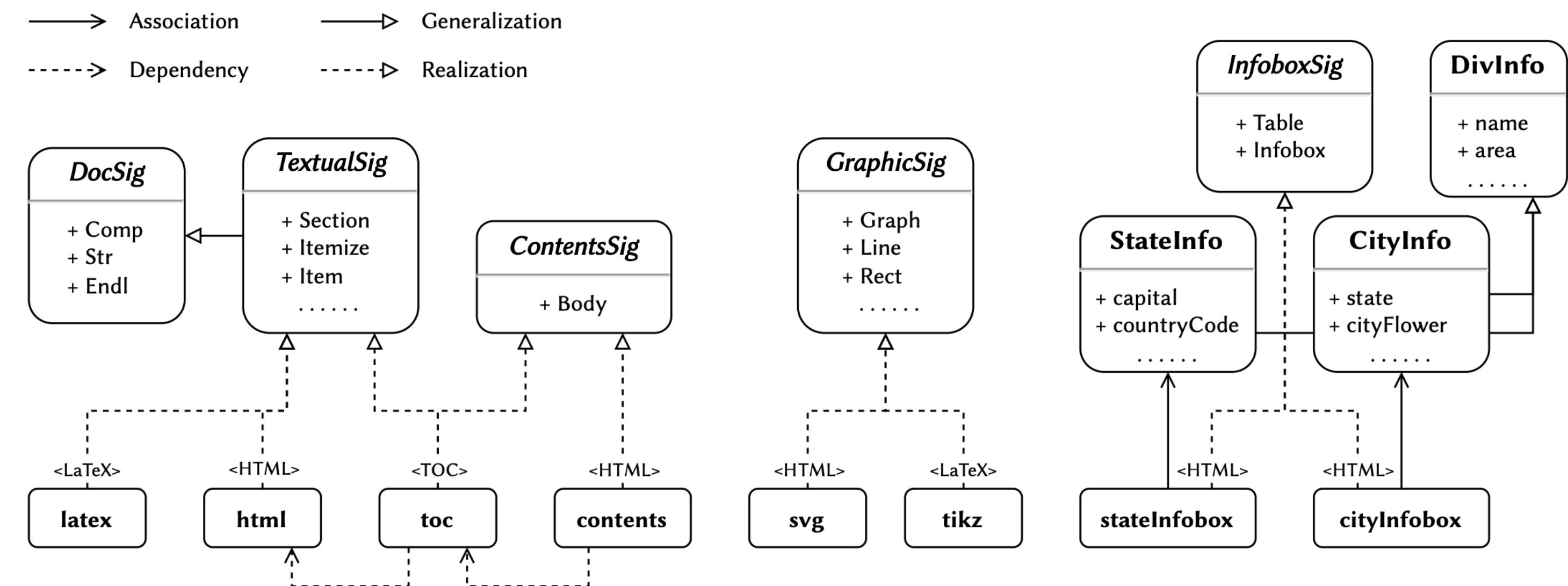
Implementation can be found in [PLGround.org](https://plground.org)

# Applications

## 1. Minipedia

No.	Country	Area (km <sup>2</sup> )	Population	Capital
1	Vatican City State	0	1000	
2	Principality of Monaco	2	38300	
3	Republic of Nauru	21	10671	Yaren
4	Tuvalu	26	11646	Funafuti
5	Republic of San Marino	61	33600	City of San Marino
6	Principality of Liechtenstein	160	38896	Vaduz
7	Marshall Islands	181	54701	Majuro
8	Federation of Saint Christopher and Nevis	261	52441	Basseterre
9	Republic of Maldives	300	383135	Malé
10	Republic of Malta	316	514564	Valletta

A simplified diagram of ExT components:



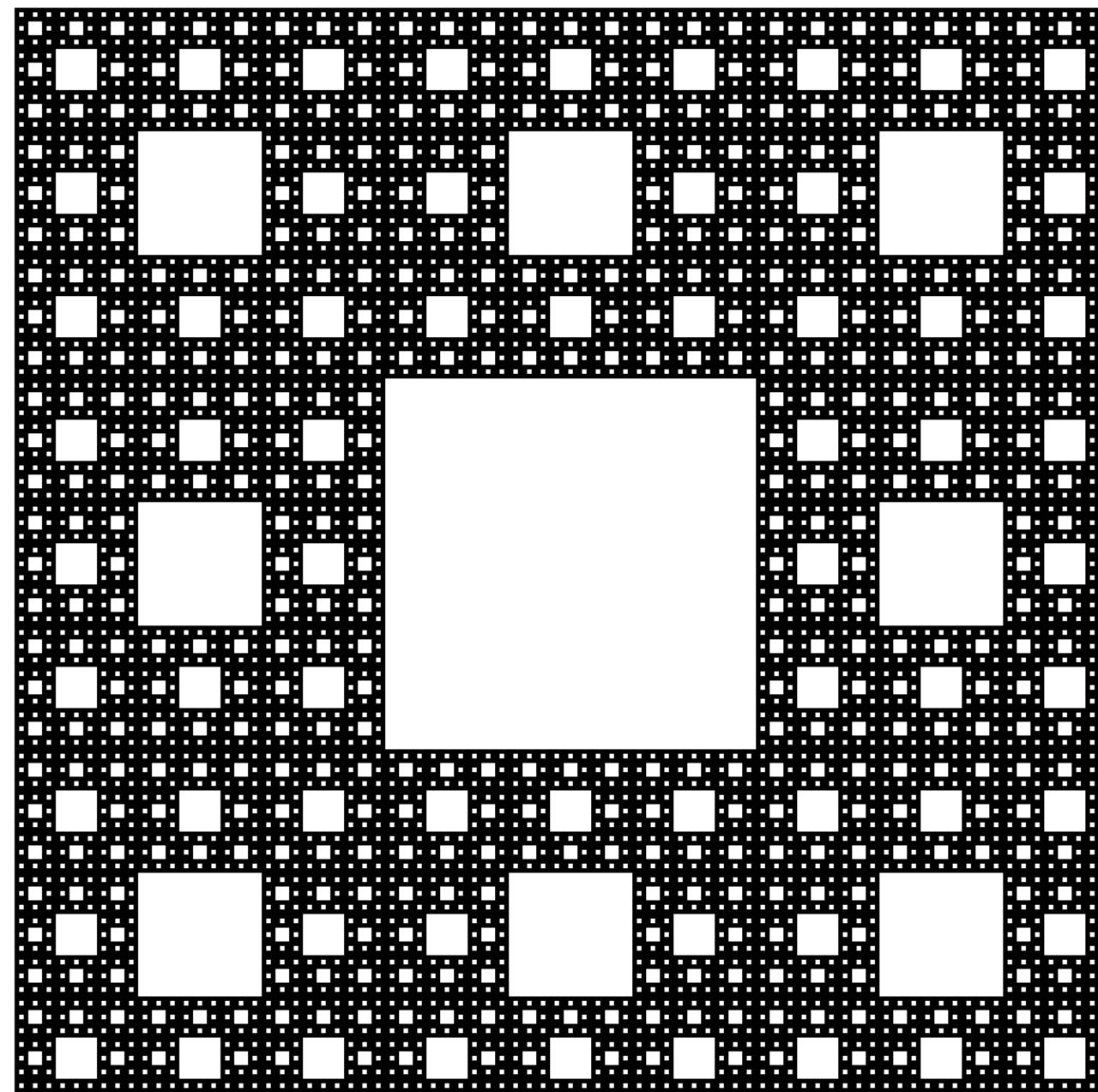
## Microstate

A **microstate** or **ministate** is a sovereign state having a very small population or very small land area, usually both. The meanings of "state" and "very small" are not well-defined in international law.<sup>[1]</sup> Recent attempts, since 2011, have not been well-defined in international law either. The term "state" is often used to refer to entities that are not states under international law, such as the Vatican City State, Monaco, and Nauru. The term "state" is also used to refer to entities that are not states under international law, such as the Marshall Islands, Federated States of Micronesia, Palau, and Tuvalu.

Case studies can also be found in [PLGround.org](http://PLGround.org)

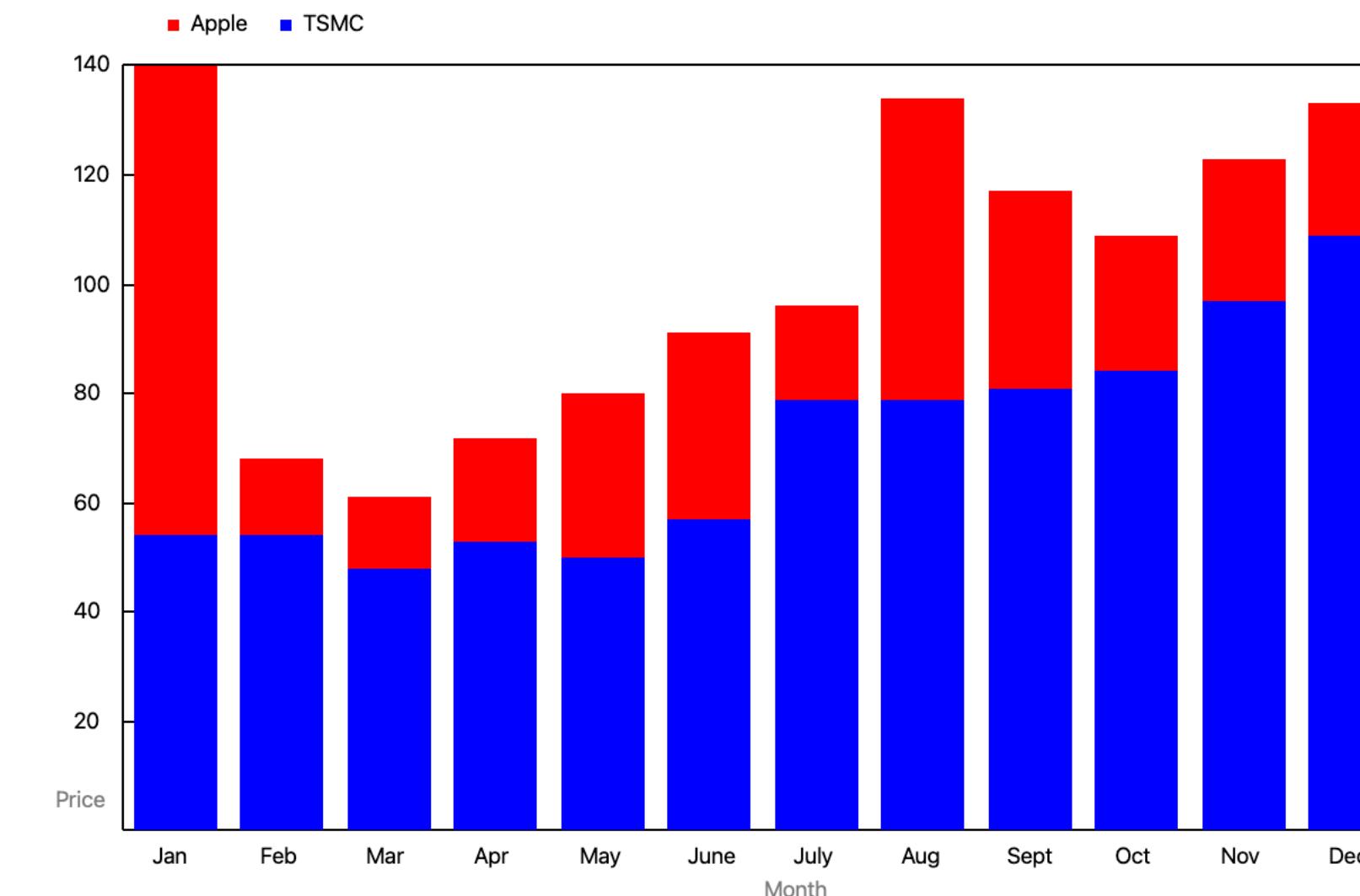
# Applications

## 2. *Fractals*



Sierpiński Carpet

## 3. *Charts*



(Both are powered by modular extensions for SVG)

Case studies can also be found in [PLGround.org](http://PLGround.org)

# Key Features of Compositional Embeddings

can also be encoded in ...

Xuan Bi, Ningning Xie, Bruno C. d. S. Oliveira, and Tom Schrijvers.  
Distributive Disjoint Polymorphism for Compositional Programming.  
In ESOP 2019.

Bruno C. d. S. Oliveira, Tijs van der Storm, Alex Loh, and William R. Cook.  
Feature-Oriented Programming with Object Algebras. In ECOOP 2013.



with a lot of boilerplate code 😞

# More in Our Paper

- Our novel way to support nested pattern matching (§3.5);
- A detailed comparison of different embeddings (§3.6);
- The design of ExT (§4);
- The complete code explaining why the tagless-final embedding cannot handle dependencies modularly (Appendix A);
- The complete code extending the tagless-final embedding to modularize dependencies with a lot of boilerplate code (Appendix B).

# Q&A